Grade 1 Mathematics, Quarter 3, Unit 3.1

Developing the Operations of Addition and Subtraction to 20

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Add and subtract fluently within 20.
- Use a variety of strategies to add and subtract within 20.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Look for an entry point to solve a problem.
- Monitor and evaluate a plan and make revisions if necessary.
- Explain and justify strategy.

Reason abstractly and quantitatively.

- Use a variety of representations to support work (i.e., diagrams, manipulatives, number lines).
- Write an equation to explain thinking or the thinking of others.
- Explain the relationship between the equations and actions taken to solve a problem.

Essential questions

- How can the strategy of a set of ten help to solve this problem (for sets between 11–20)?
- What strategy will you use to solve this problem?
- How can you explain your actions with an equation?
- What other ways can you show your thinking (i.e., diagrams, pictures, etc.)?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 1.OA

Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, ~~and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*~~

Number and Operations in Base Ten 1.NBT

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add ~~within 100,~~ including adding a two-digit number and a one-digit number, ~~and adding a two-digit number and a multiple of 10,~~ using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

Kindergarten students use concrete objects and acting out situations to solve addition and subtraction problems. The problems are real-world situations. Students learn the mathematical language that will support their discussion of their representations. In previous units in first grade, students learned to add and subtract fluently within 10. Students also understand that addition and subtraction have a relationship.

Current Learning

First-grade students use their knowledge of addition and subtraction to compare, represent, and solve for unknowns, and extend addition and subtraction to solve problems with two-digit numbers to 20. They learn to recognize the properties of addition and subtraction and how to use them to solve problems in an easier way. They learn to use diagrams to help solve more difficult problems and to rewrite the equation if needed.

Future Learning

In future grades, students extend the relationship of operations to division and multiplication. In grade 5, students begin to evaluate and interpret expressions (8 + 27) +2 or (6 x 30) + (6 x 7). Eventually, students extend their work to include negative numbers while the meaning of operations continues to evolve.

Additional Findings

According to *Principles and Standards for School Mathematics*, students should encounter a variety of meanings for addition and subtraction of whole numbers (p. 34).

Grade 1 Mathematics, Quarter 3, Unit 3.2

Using Properties of Operations to Solve Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use a variety of strategies to add and subtract within 20.
- Understand the relationship between addition and subtraction.
- Recognize how the operations can be used as strategies.
- Determine if both sides of the equal sign have the same value.

Mathematical practices to be integrated

Look for and make use of structure.

- Look for shortcuts (making a set of 10).
- Use a variety of representations to support your work (i.e., diagrams, manipulatives, number lines).

Look for and express regularity in repeated reasoning.

- Write an equation to explain one's own thinking or the thinking of others.
- Explain the relationship between equations and actions.
- Use properties of operations (3 + 7 = 7 +3)

Essential questions

- How can the strategy of a set of ten help to solve this problem (for sets between 11–20)?
- What strategy will you use to solve this problem?
- How can you explain your actions with an equation?
- What other ways can you show your thinking? (i.e., diagrams, pictures, etc.)

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 1.OA

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.3 Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

[3] Students need not use formal terms for these properties.

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Kindergarten students use concrete objects and acting out situations to solve addition and subtraction problems. The problems are real-world situations. Students learn the mathematical language that will support their discussion of their representations. In previous units in first grade, students have learned to add and subtract fluently within 10.

Current Learning

First-grade students use their knowledge of addition and subtraction to compare, represent, and solve for unknowns, and extend addition and subtraction to solve problems with two-digit numbers to 20. They learn to recognize the properties of addition and subtraction and how to use them to solve problems in an easier way. They learn to use diagrams to help solve more difficult problems and to rewrite the equation if needed.

Future Learning

In future grades, students will extend the relationship of operations to division and multiplication. In grade 5, students will begin to evaluate and interpret expressions (8 + 27) + 2 or (6 x 30) + (6 x 7). Eventually, students extend their work to include negative numbers while the meaning of operations continues to evolve (According to the *Progressions for Common Core Standards for Mathematics*).

Additional Findings

According to *Principles and Standards for School Mathematics*, students should encounter a variety of meanings for addition and subtraction of whole numbers (p. 34).

Grade 1 Mathematics, Quarter 3, Unit 3.3

Using Addition and Subtraction to Find Unknown Addends or Subtrahends

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Determine the unknown number that makes the equation true.
- Use addition problems to solve subtraction problems (i.e., fact families).
- Represent the problem situation.
- Link equations to concrete drawings, equations, and other concrete representations.
- Manipulate equations using operations and/or relationships between operations (fact families).

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Decontextualize the situation.
- Represent symbolically.
- Manipulate the symbols.

Construct viable arguments and critique the reasoning of others.

- Construct and justify conclusions using concrete objects, drawings, diagrams, and actions.
- Communicate solutions to others.
- Listen/read the arguments of others.
- Ask questions for clarification.

Attend to precision.

- Accurately compute operations.
- Use a variety of tools to check for accuracy.

Essential questions

- How can you show this problem using concrete objects, drawings, and equations?
- What are the different ways that you could find a solution?
- What other equations could you write to make this equation true?
- How can you use fact families to check your answer?
- How can you find the unknown number?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 1.OA

Work with addition and subtraction equations.

1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In kindergarten, students act out counting situations using fingers, objects, and math drawings. They translate real world situations into math terms. The relationship between addition and subtraction is that of reversibility. Students can compose a number and undo that composition as a decomposition and vice versa. They work with two addends unknown as they explore compositions as they make each number.

Current Learning

In grade 1, students work with more difficult "algebraic" problem in which a situational equation does not immediately lead to an answer. To solve, students can write a related equation called a solution equation, better known as a fact family. For example 8 + ? = 14 can be solved if I know that 14 – 8 = 6.

Routines: Students will develop fluency for addition and subtraction through 10 by daily practice.

Future Learning

Students will master all the problem situations and all of their subtypes and language variants. The numbers in the problems for addition and subtraction will be within 100. They will represent these problems with diagrams and/or equations.

Additional Findings

According to *Principles and Standards for School Mathmatics*, students should encounter a variety of meanings for addition and subtraction of whole numbers. (p. 34)

Grade 1 Mathematics, Quarter 3, Unit 3.4

Solving Addition and Subtraction Word Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Identify the appropriate strategy to use in different addition and subtraction situations (add, to, take from, put together, take apart, and compare).
- Use the relationship among the operations to check for precision.
- Recognize that an unknown can be an addend, subtrahend, sum, or a difference.
- Recognize and interpret symbols (=, +, -) used in the equation.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Make connections between the problem and the mathematical situations*.
- Plan a solution.
- Use concrete objects or pictures to conceptualize and solve.
- Explain the correspondence between the equation and the representation.
- Simplify the original problem and explain its intent.

**Situations mean add together, pull apart, take from, and compare.*

Model with mathematics.

- Write an addition/subtraction equation to model.
- Apply knowledge about addition and subtraction.
- Make any necessary revisions.

Attend to precision.

- Use of tools to check for accuracy.

Essential questions

- What information does this word problem give us?
- What do we need to find out?
- What is the plan to solve the problem?
- How can you show this problem using concrete objects, drawings, and equations?
- What is the strategy to solve the problem?
- What are the different ways that you could find a solution?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

NOTE: [2] See Glossary, Table 1.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Kindergartners are adding and subtracting real world situations using their finger, objects, and drawings. The students are using a combination of mathematical and non-mathematical language to explain their representations and solutions. Teachers write the representations as well as the equations to represent the whole situation. Kindergarteners have to add to, take from, put together, take apart, compose a number to assist them with problems through ten. By the end of kindergarten they were able to solve equations through 5. In kindergarten, they began to recognize the relationship between addition and subtraction.

Current Learning

In grade 1, students begin to represent a new type of problem situation called comparing. They need extensive experience to master the language and complexities of the situation. They read to understand the problem situation, represent it and manipulate the representation to solve it. Students begin to relate equation to diagrams to understand and solve more complex problems. Routines: Students will develop fluency for addition and subtraction through 10 by daily practice.

Future Learning

In grade 2, students begin to use 2-step word problems. They will begin to use addition and subtraction within 100 to solve problems. By the end of the K-2, they have enough experience to add and subtract single digit sums from memory. In grades 2 and 3, they extend their situations to deal with length.

Additional Findings

According to A Research Companion to Principles and Standards for School Mathematics, students have learned to compute and then apply those computations. Research has indicated that beginning with problem situations yields greater problem-solving competence and equal or better computational competence (p. 68). Even kindergarten students can solve many of these problems if they use objects to directly model the situation (p. 69). The most powerful problem-solving approach is to understand the situation deeply—that is, to be able to draw it or otherwise represent it to oneself (p. 69).